

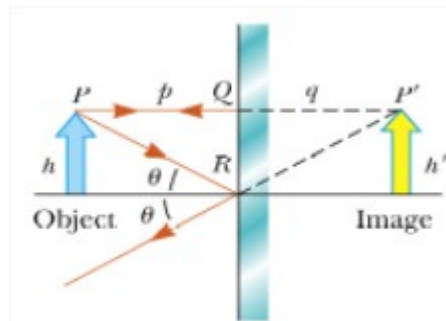
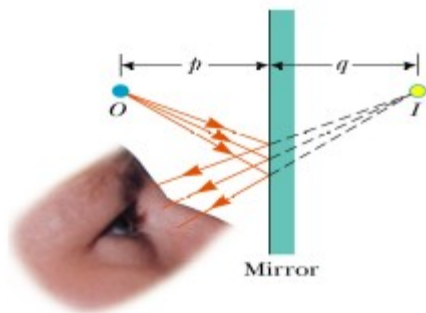
# Mirrors and Lenses

In this chapter we will build upon the laws of reflection and refraction to understand how images are formed by mirrors and lenses.

## Mirrors

### Flat Mirrors

A flat mirror will produce an image that is behind the mirror. The image is the same distance from the mirror as the object and is upright if the object is upright. This is a consequence of the law of reflection and a bit of geometry.



The object distance is  $p$  and the image distance is  $q$ . For flat mirror,  $p = q$

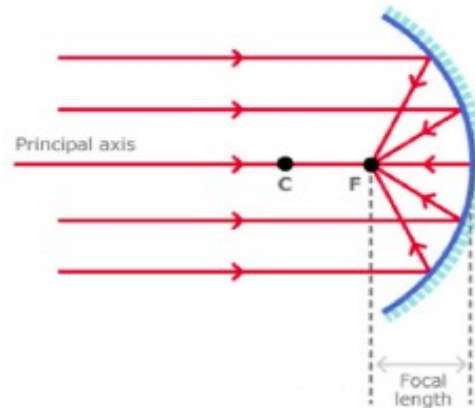
The magnification is the ratio of the image height to the object height.

$$M = \frac{\text{image height}}{\text{object height}} = \frac{h'}{h}$$

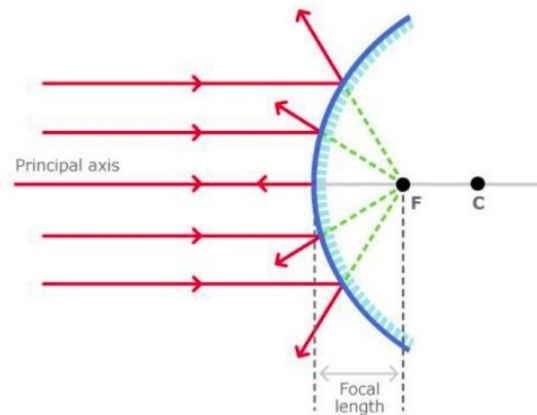
For flat mirror,  $M = 1$ .

### Concave Mirror (converging)

A spherical mirror has the shape of a segment of a sphere. The mirror is characterized by its radius of curvature  $R$ , and center of curvature  $C$ . The mirror axis (or principal axis) passes through  $C$  and the vertex  $V$  of the mirror.



### Convex Mirror (diverging)



### **Ray Diagrams for Mirrors**

If the object position and size are known, then the image position and size can be determined graphically by constructing ray diagrams. For any point which belong to the the objects there is a corresponding point which belong to the image. The image point is found by the intersection of the ray diagrams . The image of an object is then constructed once enough image points are obtained. In most cases, as for example if the object is simply an arrow, two images points are sufficient to find the image, one for each end of the arrow.

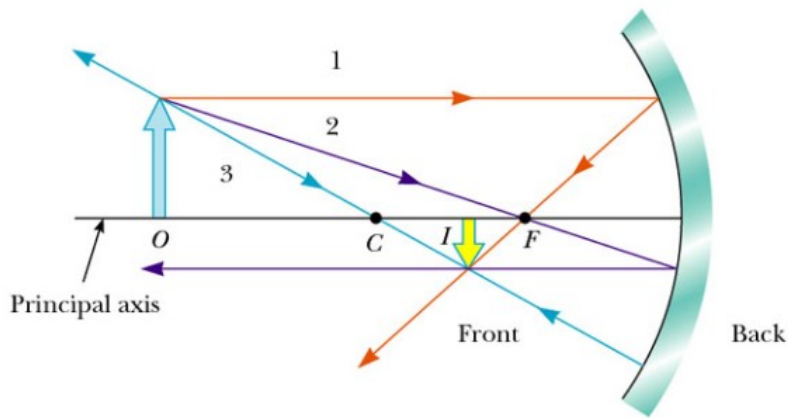
The rules for drawing the rays are as follows:

1. A ray parallel with the mirror axis is reflected through the focal point. (This follows from the definition of the focal point.)
2. A ray passing through the focal point is reflected parallel with the mirror axis. (This follows from the reversibility of the rays.)
3. A ray passing through the center of curvature is reflected back through the center of curvature. (This is because the radius is perpendicular to the mirror surface.)

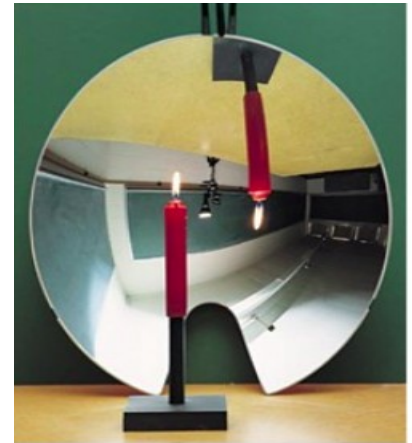
The rules above apply for any mirror, but we need to distinguish three separated cases

### Concave Mirror

1. The rays below are drawn for the case when the object is more far away to the mirror than the focal point.



$$p > f$$

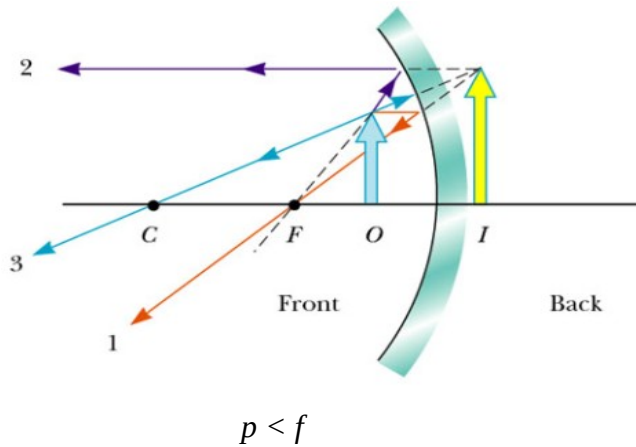


As can be seen from the diagram, the image is inverted and reduced in size.

In this case the rays go through the image, the image is said to be *real*.

It's called real in the sense that the image can be projected on a screen (like the image on a movie theater) and you can actually draw with a pencil over it.

2. The rays below are drawn for the case when the object is between the focal point and the mirror.

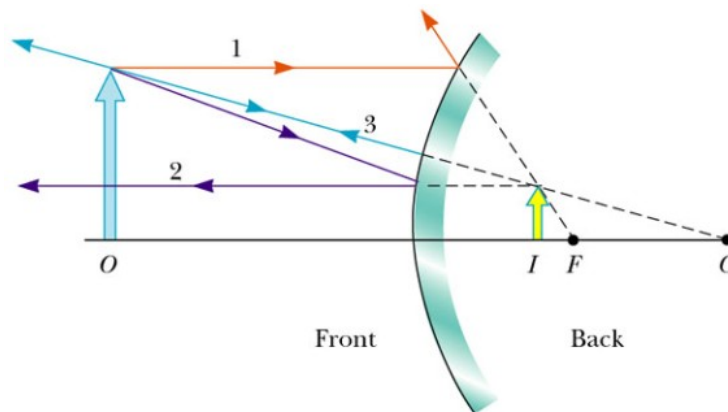


In this case, however, the reflected rays don't all intersect at a point in front of the mirror. Instead, we must project the rays behind the mirror to find an intersection and the position of the image. In this case, the image is upright and enlarged. Since the rays of light don't actually go through the image, the image is said to be *virtual*. You can see this image, but it cannot be projected on a screen. If you had a pencil you could not draw over it (the image exists only inside the mirror). The image is upright.

Flat mirror also produce a virtual image: the image of yourself in the mirror is virtual, it exists only inside the mirror.

### Convex Mirror

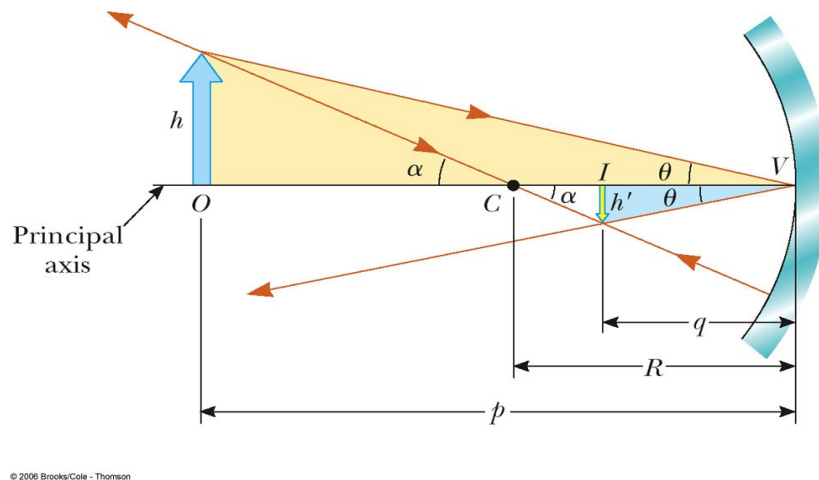
3. A convex mirror has its center of curvature and focal point on the back side of the mirror. The figure below shows ray diagrams for a convex mirror with the object in front of the mirror. The rays are drawn using the same basic rules as before. However, we must extrapolate the reflected rays to the focal point and to the center of curvature at the back side of the mirror.



By extrapolation to the back side of the mirror, the reflected rays intersect to define the image on the back side. The image is upright and reduced in size. It is virtual since the reflected rays do not actually pass through the image.

### Mirror Equations

The construction below shows two rays emanating from the tip of the object. One ray strikes the vertex of the mirror at an angle  $\theta$  and is reflected at the same angle  $\theta$ . The other ray goes through the center of curvature and towards the mirror along the mirror radius. It strikes the mirror at perpendicular incidence and is reflected back along the same path. The intersection of the two reflected rays defines the tip of the image arrow. The object distance is  $p$  and the image distance is  $q$ .



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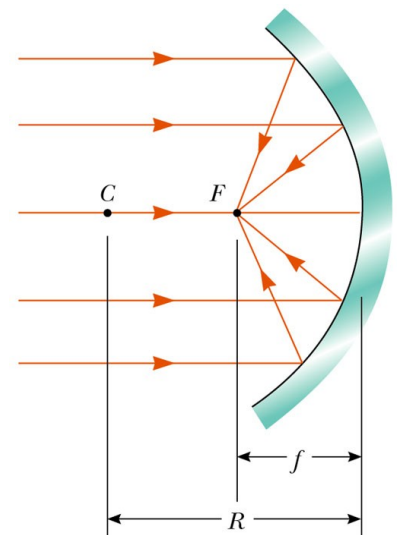
The two shaded triangles defined by the ray striking the mirror vertex are ‘similar’, so that

$$M = \frac{h'}{h} = -\frac{q}{p}$$

In this particular construction,  $p$  and  $q$  are both positive.  $h$  is positive and  $h'$  is negative (inverted), thus the minus sign in the equation. For this particular geometry,  $M$  is negative and  $|M| < 1$ .

The ray going through  $C$  intersects the mirror axis at an angle  $\psi$ . It also forms two similar triangles with  $h$  and  $h'$  being one of the sides. From these two triangles and a bit of algebra, one can obtain the *mirror equation*

$$\frac{1}{p} + \frac{1}{q} = \frac{2}{R}$$



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The mirror equation enables one to calculate the image position from the object position and the radius of curvature of the mirror.

A spherical mirror can also be characterized by its **focal length**. The focal length is the image distance when the object distance is infinity. From the mirror equation, we see that as

$$p \rightarrow \infty \Rightarrow \frac{1}{q} = \frac{1}{f} = \frac{2}{R}$$

Thus, the focal length is

$$f = \frac{R}{2}$$

The focal point  $F$  is the point of intersection of the reflected rays when the incoming rays originate from far away and are essentially parallel.

Thus, we can also write the mirror equation as

$$\frac{1}{p} + \frac{1}{q} = \frac{1}{f}$$

### Sign conventions for mirrors

In applying the mirror equation, it is important to take note of sign conventions:

- If the object is on the front side of the mirror, it is *real* and the object distance is positive. If the object is on the back side, it is *virtual* and the object distance is negative. (Note: An object can appear to be on the backside if the incoming rays of light converge while approaching the mirror.)
- If the image is on the front side of the mirror, it is real and the image distance is positive. If it is on the back side, it is virtual and the image distance is negative.
- If the center of curvature is on the front side (as for a concave mirror), then the radius of curvature and the focal length are positive. If it is on the back side (as for a convex mirror), then the radius of curvature and the focal length are negative.
- If the image is upright, then its height is positive and the magnification is positive. If the image is inverted, then its height is negative and the magnification is negative.

### Sign Convention for Mirror

Quantity	Symbol	In Front	In Back
Object location	$p$	+	-
Image location	$q$	+	-
Focal length	$f$	+	-

The table below shows the proprieties of the image in relation to its magnification.

### Magnifications Table

$M < -1$	$-1 < M < 0$	$0 < M < 1$	$1 < M$
Real	Real	Virtual	Virtual
Bigger	Smaller	Smaller	Bigger

### *Example*

An object 2 cm high is placed 30 cm in front of a concave mirror with radius of curvature 20 cm. Calculate the image position, magnification, and height of the image.

$$\frac{1}{p} + \frac{1}{q} = \frac{2}{R}$$
$$\frac{1}{q} = \frac{2}{R} - \frac{1}{p} = \frac{2}{20 \text{ cm}} - \frac{1}{30 \text{ cm}}$$
$$q = 15 \text{ cm}$$
$$M = -\frac{q}{p} = -\frac{15}{30} = -0.5 = \frac{h'}{h}$$
$$h' = -0.5h = -0.5(2 \text{ cm}) = -1 \text{ cm}$$

The image is real ( $q > 0$ ), inverted ( $M < 1$ ), and reduced in size.

*Example*

Repeat the above example if the object is placed 5 cm from the mirror.

$$\begin{aligned}\frac{1}{q} + \frac{1}{R} &= \frac{1}{p} = \frac{1}{20 \text{ cm}} - \frac{1}{5 \text{ cm}} \\ q &= -10 \text{ cm} \\ M &= -\frac{q}{p} = -\frac{-10}{5} = 2 = \frac{h'}{h} \\ h' &= 2h = 2(2 \text{ cm}) = 4 \text{ cm}\end{aligned}$$

So, the image is virtual ( $q < 0$ ), upright ( $M > 0$ ), and enlarged. It is on the backside of the mirror.

*Example*

A 3-cm object is placed 5 cm in front of a convex mirror of radius focal length 10 cm. Find the image position, magnification, and image size.

Since the mirror is convex, then the focal length is negative.

$$\begin{aligned}\frac{1}{p} + \frac{1}{q} &= \frac{2}{R} = \frac{1}{f} \\ \frac{1}{q} &= \frac{1}{f} - \frac{1}{p} = \frac{1}{-10 \text{ cm}} - \frac{1}{5 \text{ cm}} \\ q &= -3.3 \text{ cm} \\ M &= -\frac{q}{p} = -\frac{-3.3}{5} = 0.67 = \frac{h'}{h} \\ h' &= 0.67h = 0.67(3 \text{ cm}) = 2 \text{ cm}\end{aligned}$$

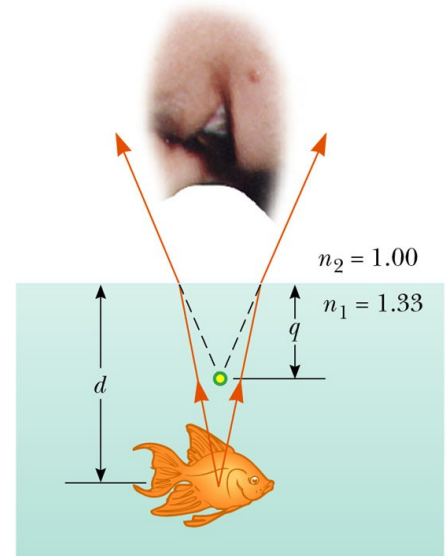
The image is virtual ( $q < 0$ ), it is upright ( $M > 0$ ), and reduced. It is located on the backside of the mirror.

In all of the above examples it is strongly recommended that you also draw a ray diagram to approximate the image position and magnification as a check on your calculations.



## Images formed by a single refracting surface

If an object in one medium is viewed from another medium, then the image of the object will be affected by the refraction of light at the interface between the two media. For example, a fish under water when viewed from above will appear to be closer to the surface that it really is. The curvature of the surface will also have an effect. The image of a fish in a bowl of water will depend on the radius of curvature of the bowl as well as the index of refraction of the water.



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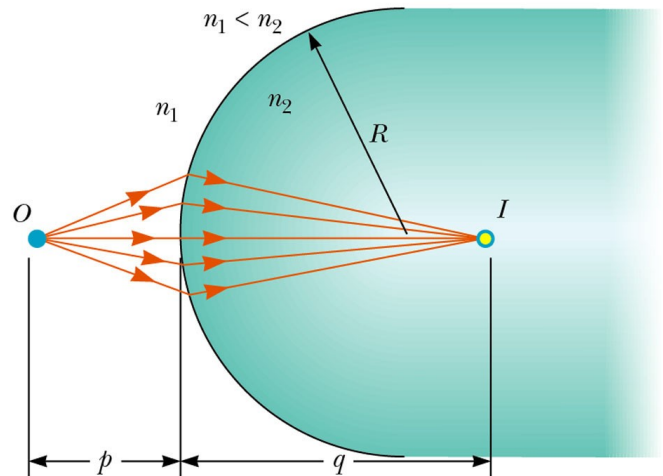
Consider the construction below in which the interface between two media is a spherical surface. This could, for example, represent a glass rod of index of refraction  $n_2$  with a rounded end surrounded by air ( $n_1 = 1$ ).

If the object is not too close to the surface, the light entering the rod will be refracted and form an image inside the rod. Using Snell's law and a bit of geometry, it can be shown that the object distance,  $p$ , and image distance,  $q$ , are related by the equation

$$\frac{n_1}{p} + \frac{n_2}{q} = \frac{n_2 - n_1}{R}$$

In the above example  $p$ ,  $q$ , and  $R$  are all considered positive. If the center of curvature is on the object side, then  $R$  is negative. The image above is real and the image distance is positive. If the image is on the object side, then it is virtual and the image distance is negative. A virtual image would exist if the object is sufficiently close to the surface or if  $n_2 < n_1$ .

The magnification for a single refracting surface is given by

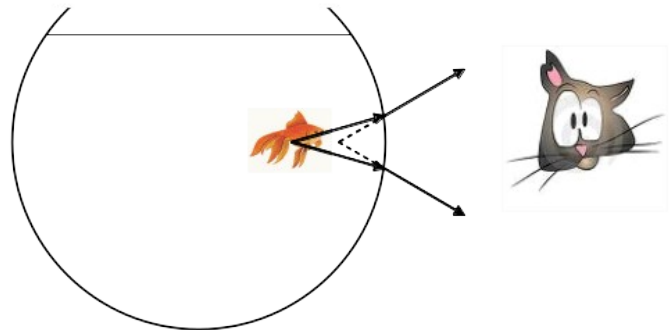


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$$M = -\frac{n_1 q}{n_2 p}$$

*Example*

A goldfish in a bowl of water is 10 cm from the side. The radius of the bowl is 20 cm. Where is the image of the goldfish as seen from an observer outside the bowl?



$n_1 = 1.33$ ,  $n_2 = 1$ ,  $R = 20$  cm,  $p = 10$  cm

$$\frac{1.33}{10} + \frac{1}{q} = \frac{1 - 1.33}{20}$$

$$\frac{1}{q} = -0.1495$$

$$q = -6.69 \text{ cm}$$

Since  $q$  is negative, the image is virtual and is on the same side of the surface as the object.

The magnification of the goldfish is

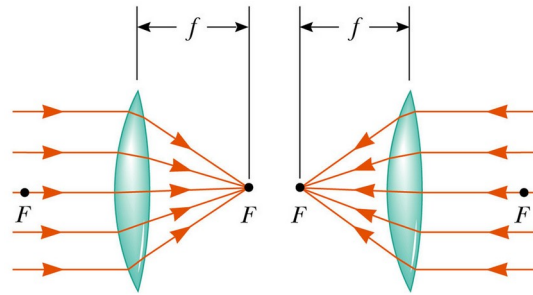
$$M = -\frac{n_1 q}{n_2 p} = -\frac{(1.33)(-6.69)}{(1)(10)} = 0.89$$

So, the image is upright and reduced in size.

## Lenses

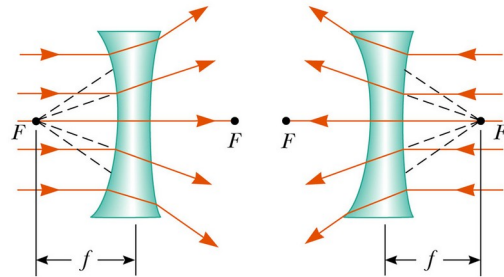
A lens consists of two curved refracting surfaces. The image formed by a lens depends on the index of refraction of the lens and the curvature of each of the surfaces. A lens has a focal point on each side of the lens and a focal length. The focal point is the image position when the object is very far away. That is, it is the point where incoming parallel rays intersect (either directly, or by extrapolation) after they pass through the lens. A *converging* lens has a positive focal length and the image formed by a far away object is real and on the opposite side of the lens from the object. A *diverging* lens has a negative focal length and the image formed by a far away object is virtual and on the same side as the object.

### Convex lens (converging)



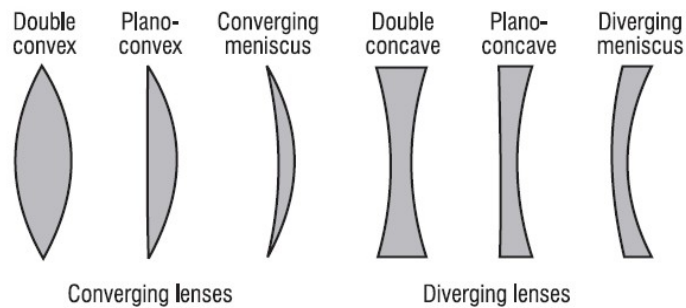
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### Concave lens (diverging)



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If a lens is thicker on the principal axis then is converging (and vice versa)



## Ray Diagrams for Lenses

We can geometrically find an image by drawing principle rays, similar to what can be done for a mirror.

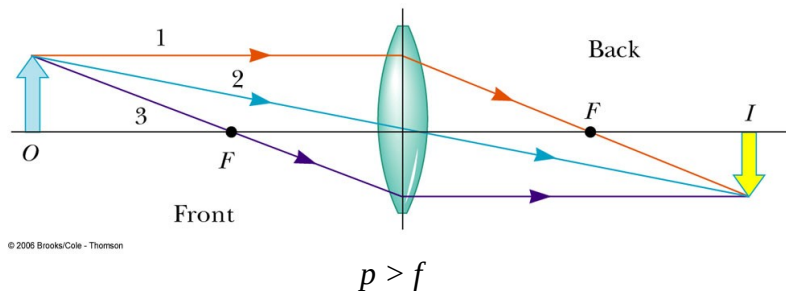
1. A parallel ray going from the object to the lens passes through the focal point. For a converging lens, this will be the focal point opposite the object. For a diverging lens, this will be, by extrapolation backwards, the focal point on front side of the lens.
2. A ray from the object through the center of the lens will continue in the same direction..
3. A ray passing going from the object to the lens that passes through the focal point will emerge parallel. For a converging lens, the focal point will be on the front side. For a diverging lens, the focal point will be, by extrapolation of the incoming ray, on the back side.

The image is determined by the intersection of the rays that pass through the lens (either directly, for a real image, or by back extrapolation, for a virtual image).

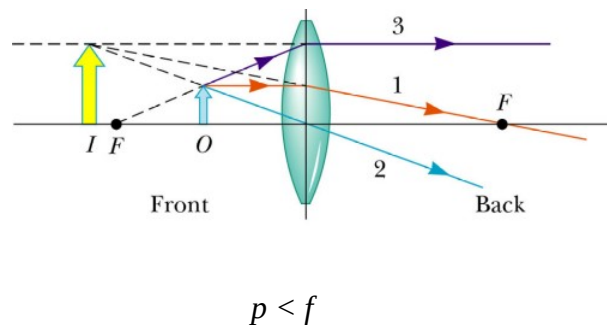
As for mirrors, we need to distinguish three separated cases

### Convex lens

1. Below are principle rays drawn for a converging lens when  $p > f$ . The image is real and inverted.

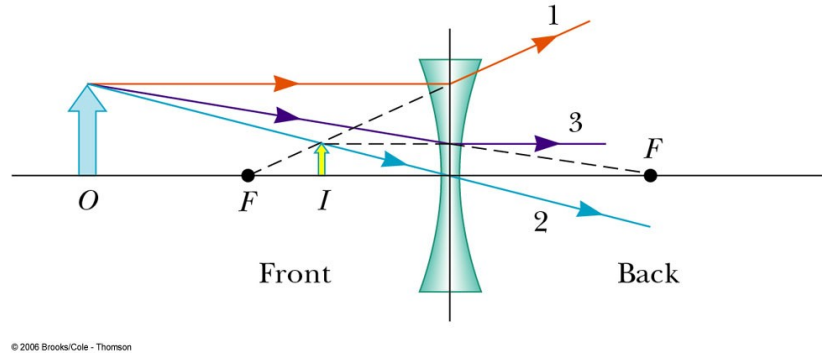


2. If the object is inside the focal point of a converging lens, then the image will be virtual and upright, as shown below.



## Concave lens

3. Ray diagrams for a diverging lens are shown below.



## **Lens Equations**

It can be shown that the object and image distances for a thin lens are related by an equation that is exactly the same as for a spherical mirror,

$$\frac{1}{p} + \frac{1}{q} = \frac{1}{f}$$

However, the focal length is given by the *lens maker's equation*,

$$\frac{1}{f} = (n-1) \left( \frac{1}{R_1} - \frac{1}{R_2} \right),$$

$R_1$  is the radius of curvature of the first surface and  $R_2$  is the radius of curvature of the second surface (defined with respect to the incoming rays of light).

## Sign conventions for lenses

- If the object is on the front side of the lens, it is *real* and the object distance is positive. If the object is on the back side, it is *virtual* and the object distance is negative. (Note: An object can appear to be on the backside if the incoming rays of light converge while approaching the lens.)
- If the image is on the back side of the lens, it is real and the image distance is positive. If it is on the front side, it is virtual and the image distance is negative. (Note: The rule is opposite to that for a mirror.)

- If the center of curvature of a lens surface is on the back side of the lens, then  $R$  is positive. If it is on the front side, then  $R$  is negative.
- If the image is upright, then its height is positive and the magnification is positive. If the image is inverted, then its height is negative and the magnification is negative.

Sign Convention for Lenses

Quantity	Symbol	In Front	In Back
Object location	$p$	+	-
Image location	$q$	-	+
Focal length	$f$	+	-

The table below shows the proprieties of the image in relation to its magnification.

Magnifications Table

$M < -1$	$-1 < M < 0$	$0 < M < 1$	$1 < M$
Real	Real	Virtual	Virtual
Bigger	Smaller	Smaller	Bigger

*Example*

The radii of curvature of the surfaces of a convex-convex lens are 20 cm and 30 cm. The index of refraction of the lens is 1.6. Find the focal length.

If the curvature of the first surface is 20 cm, then  $R_1 = 20$  cm and  $R_2 = -30$  cm. Applying the lens maker's equation, we have

$$\frac{1}{f} = (n-1) \left( \frac{1}{R_1} - \frac{1}{R_2} \right) = (1.6-1) \left( \frac{1}{20} - \frac{1}{-30} \right) = 0.05$$

$$f = 20 \text{ cm}$$

*Example*

An object is placed 30 cm in front of a lens with focal length 20 cm. Find the object position and the magnification.

$$\frac{1}{q} = \frac{1}{f} - \frac{1}{p} = \frac{1}{20} - \frac{1}{30} = 0.0167$$
$$q = 60 \text{ cm}$$
$$M = -\frac{q}{p} = -\frac{60}{30} = -2$$

The image is real, on the back side of the lens, inverted, and enlarged by a factor of 2.

*Example*

A 3-cm tall image is placed 10 cm in front of a lens of focal length 25 cm. Find the position and size of the image.

$$\frac{1}{q} = \frac{1}{f} - \frac{1}{p} = \frac{1}{25} - \frac{1}{10} = -0.06$$
$$q = -16.7 \text{ cm}$$
$$M = -\frac{q}{p} = -\frac{-16.7}{10} = 1.67$$
$$h' = Mh = (1.67)(3 \text{ cm}) = 5 \text{ cm}$$

The image is virtual, on the same side as the object, upright, and enlarged.

*Example*

An object is placed 15 cm in front of a diverging lens of focal length -5 cm. Locate the image.

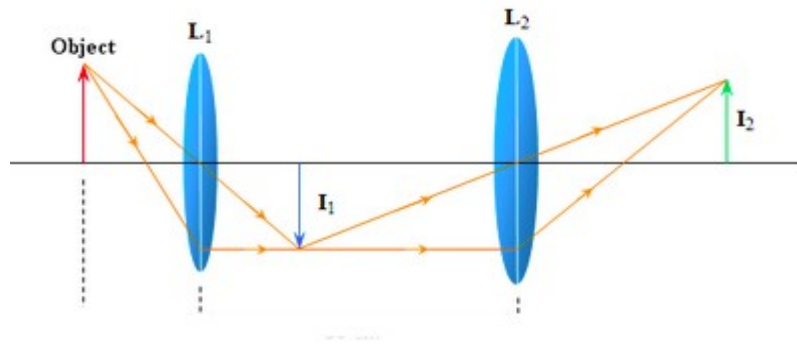
$$\frac{1}{q} = \frac{1}{f} - \frac{1}{p} = \frac{1}{-5} - \frac{1}{15} = -0.267$$
$$q = -3.75 \text{ cm}$$
$$M = -\frac{q}{p} = -\frac{-3.75}{15} = 0.25$$

The image is virtual, on the same side as the object, upright, and reduced in size.

In all the above examples, it is important to be able to qualitatively verify the numerical calculations by drawing a ray diagram.

## Lens combinations

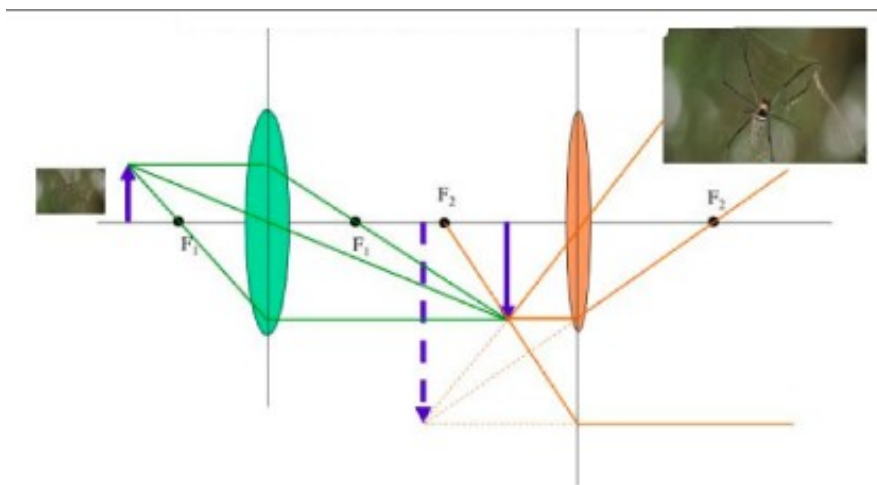
If two lenses are used in sequence, then the final image is obtained as follows. Find the image due to the first lens. Find the location of this image from the 2<sup>nd</sup> lens, and use this image as the object for the 2<sup>nd</sup> lens. The overall magnification will be the product of the magnification of each lens,  $M = M_1 M_2$ .



The image of the first lens  $I_1$  becomes the object for the second lens  $O_2$

## Telescope/microscope

When  $I_1$  is formed at a distance from the second lens shorter than  $f_2$  then the image  $I_2$  is virtual (it appears inside the microscope) and is larger than  $O_1$

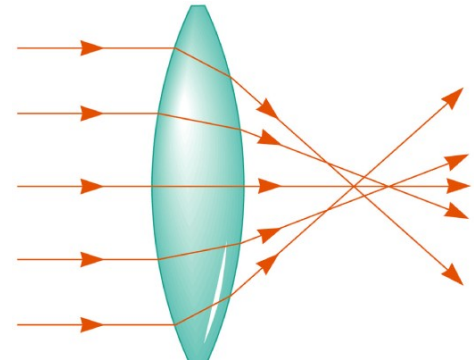




## Spherical aberration

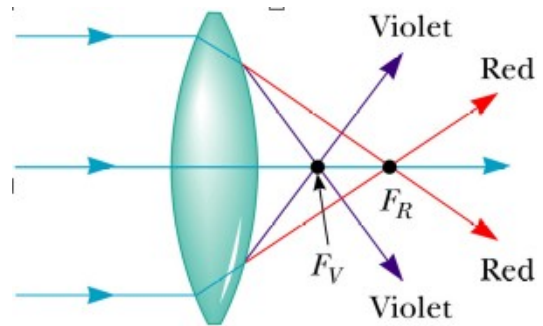
The previous discussions assumed that there was a well-defined image that would be produced by all rays of light.

In reality, lenses have limitations that will cause image distortions. If one assumes that the surfaces of a lens are spherical, then the rays that are far above and below the principle axis of the lens will be refracted more than those near the center of the lens. This effect, called *spherical aberration*, will cause the image to be blurred. Spherical aberration can be reduced by limiting the light to the center part of the lens.



## Chromatic aberration

If white light is used, then *chromatic aberration* will exist. This is because the index of refraction, and thus the focal length, depends on wavelength and color. As a consequence the image of a white object will be slightly spread into colors.



There are technical ways of reducing chromatic aberration, including the use of compound lenses consisting of converging and diverging lenses.

Using a second diverging lens the rays focus on a single point.

